

Modeling and Energy–Optimal Neuro–Fuzzy Control of Cylindrical Autonomous Underwater Vehicles

Nguyen Minh Tan^{ID}, Vuong Duc Phuc*^{ID}, and Nguyen Quang Trung^{ID}

Abstract: This paper presents a reduced–order mathematical model and an energy–optimal neuro–fuzzy control method for a cylindrical autonomous underwater vehicle (AUV). The standard six DOFs formulation is simplified to translational motions under the assumption that rotational motions are stabilized near a nominal operating point. To address the concern that a diagonal reduction may become three independent single DOF systems, cross–axis coupling terms are retained in the reduced inertia and damping matrices. The vehicle is represented with four thrusters acting on three translational degrees of freedom; therefore, the control–effectiveness matrix is rectangular and the actuator–force vector is not unique for a given virtual control force. The proposed controller combines adaptive neuro–fuzzy uncertainty compensation with a quadratic actuator–allocation layer that penalizes force mismatch, propulsion effort and command variation. Illustrative MATLAB/Simulink–style simulations compare the method with PID, fuzzy–PID and sliding mode control using tracking error, energy index and actuator–saturation metrics.

Keywords: Actuator allocation, autonomous underwater vehicle, coupled three–DOF model, energy optimization, MATLAB/Simulink, neuro–fuzzy control.

1. INTRODUCTION

AUVs have nonlinear and coupled dynamics and are strongly affected by added mass, hydrodynamic damping, buoyancy, environmental disturbances and actuator constraints. A complete six–DOF model is useful for physical interpretation, but a reduced control–oriented model is often preferable during an initial simulation stage because it lowers computational cost and makes controller assessment more transparent [1]–[6].

The platform considered in this study is a cylindrical or torpedo–shaped AUV–like body intended for motion–control research. The scope is limited to modeling, trajectory tracking and energy–oriented actuator command generation. Mission–level guidance, target engagement or weapon–related functions are outside the scope of this work. The rotational degrees of freedom are assumed to remain stable near an operating point by passive restoring characteristics or by a lower–level attitude–stabilization loop.

Classical PID, fuzzy–PID, sliding–mode and backstepping controllers have been widely applied to underwater vehicles. PID control is simple but can be sensitive to hydrodynamic uncertainty. Sliding–mode

control improves robustness, but it may increase command variation when the switching term is not smoothed [8], [9]. Neuro–fuzzy control is attractive because fuzzy rules can describe local nonlinear behavior while adaptive weights can learn unmodeled terms online [10]–[12].

The main contribution of this revised paper is the development of a reduced yet dynamically coupled three–DOF AUV model with a redundant four–thruster configuration for energy–aware control allocation. Unlike a strictly diagonal reduction, the model retains cross–axis coupling in the inertia and damping terms. The rectangular actuator matrix also gives a clear reason for using optimization–based control allocation, because several thrust combinations can generate similar generalized translational forces [13]–[16].

The revised formulation has four specific points: (i) the reduced 3–DOF model keeps coupling terms instead of treating surge, sway and heave as three unrelated channels; (ii) the four–thruster arrangement is illustrated and linked to the effectiveness matrix; (iii) the energy proxy is explained as a relative propulsion–effort index rather than measured electrical energy; and (iv) the simulation section is reorganized with a complete

Manuscript received May 2, 2026; received in revised form June 10, 2026; accepted July 10, 2026; available online September 30, 2026.

Nguyen Minh Tan is a PhD candidate in Automation, Vietnam Maritime University, Haiphong, Vietnam (e–mail: nguyenminhtan.ktqs@gmail.com). Vuong Duc Phuc is with the Faculty of Electrical and Electronics Engineering, Vietnam Maritime University, Haiphong, Vietnam (e–mail: phucvd.ddt@vimaru.edu.vn), Haiphong, Vietnam. Nguyen Quang Trung is with the Institute of Naval Engineering, Haiphong, Vietnam (e–mail: nguyenquangtrung27041976@gmail.com).

* Corresponding author.

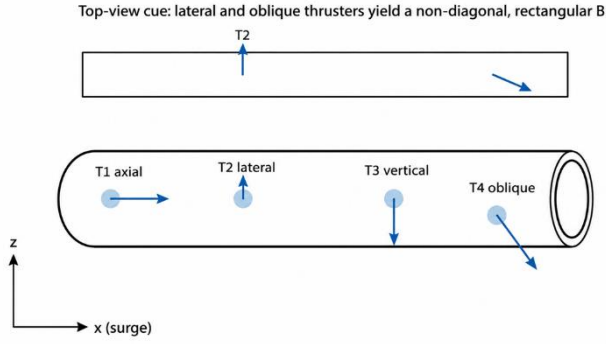


Fig. 1. Cylindrical AUV and representative four-thruster arrangement.

Table 1. Representative thruster directions and unit effectiveness vectors b_i used to form the rectangular matrix B .

Thruster	Direction	Direction vector b_i	Main role
T_1	Axial/ster n channel	$[1.00, 0.00, 0.00]^T$	Surge
T_2	lateral channel	$[0.00, 1.00, 0.00]^T$	Sway
T_3	vertical channel	$[0.00, 0.00, -1.00]^T$	Heave
T_4	oblique auxiliary channel	$[0.45, -0.25, -0.86]^T$	Redundant coupled component

Table 2. Nominal parameters for the reduced 3-DOF AUV model.

Symbol	Value	Description
L, D	1.80m, 0.22m	Cylindrical-body length and maximum diameter
m_x, m_y, m_z	62, 78, 82 kg	Effective diagonal masses
m_{xy}, m_{yz}	1.5, 2.1 kg	Representative coupling mass elements
d_x, d_y, d_z	18, 28, 32 Ns/m	Linear damping coefficients
q_x, q_y, q_z	25, 45, 50 Ns ² /m ²	Quadratic damping coefficients
$f_{\max}, \Delta f_{\max}$	65 N, 15 N/s	Maximum channel force and rate limit
T_x, T_y, T_z	0.18, 0.22, 0.25s	Actuator time constants
τ_d	< 10% of nominal thrust	Bounded current-like disturbance

parameter set, scenarios and performance indices.

A further objective is to make the simulation stage sufficiently explicit so that later experimental work can be compared with the same assumptions. For this reason, the numerical conditions are not treated as secondary implementation details. They are defined as part of the methodology, because the behavior of an energy-oriented AUV controller depends strongly on sampling time, actuator bandwidth, disturbance spectrum, initial error, saturation limits and the form of the reference

trajectory.

2. AUV DYNAMIC MODEL

2.1. General 6-DOF formulation

Let η denote the earth-fixed position and attitude vector and v denote the body-fixed velocity vector. The standard marine-craft kinematic and dynamic models are [1], [3], [6]

$$\dot{\eta} = J(\eta)v \quad (1)$$

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau + \tau_d \quad (2)$$

where, M is the inertia matrix including rigid-body and added-mass terms, $C(v)$ is the Coriolis and centripetal matrix, $D(v)$ is the hydrodynamic damping matrix, $g(\eta)$ contains restoring forces and moments, τ is the generalized control vector and τ_d is the environmental disturbance.

2.2. Translational reduction with retained coupling

For the experimental model considered here, the rotational angles are assumed to be bounded around trim values φ_0 , θ_0 and ψ_0 . Therefore, the controlled states are reduced to $\eta_3 = [x \ y \ z]^T$ and $v_3 = [u \ v \ w]^T$. This reduction follows common control-oriented AUV modeling practice in which the full vehicle dynamics are simplified around an operating condition [1], [4], [6]. The reduced kinematic relation is

$$\dot{\eta}_3 = R_0 v_3, \quad R_0 = R(\phi_0, \theta_0, \psi_0) \quad (3)$$

In the simulation coordinate frame, R_0 is approximated by the identity matrix. The transition after Eq. (3) is clarified here: the reduction removes direct attitude states from the main tracking loop, but it does not require all translational interactions to be neglected. To reflect a more realistic physical system, the reduced dynamic model includes cross-coupling effects and is expressed as

$$M_3 \dot{v}_3 + D_l v_3 + D_q(v_3)v_3 + g_3 = \tau_3 + \tau_{d3} \quad (4)$$

in which

$$D_l = \text{diag}(d_x, d_y, d_z) + D_c \quad (5)$$

while $M_3 = [m_{ij}]_{3 \times 3}$ and $i, j \in \{x, y, z\}$, and

$$D_q(v_3) = \text{diag}(q_x|u|, q_y|v|, q_z|w|) \quad (6)$$

The off-diagonal entries of M_3 and the retained cross terms in the damping representation prevent the reduced plant from becoming three unrelated single DOF systems. The model remains reduced, but it preserves the coupling that is important for a fair assessment of multi-axis control and actuator allocation.

2.3. Thruster arrangement and control effectiveness matrix

To justify control allocation, the AUV is represented with four independent thrusters controlling three translational degrees of freedom. Two channels mainly support horizontal motion, one channel is mainly vertical and an auxiliary oblique channel introduces redundancy and cross-axis force components. The schematic in Fig. 1 provides the physical interpretation of the actuator matrix.

The actuator force vector $f = [f_1 \ f_2 \ f_3 \ f_4]^T$ generates the generalized translational force $\tau_3 = [X \ Y \ Z]^T$ through the

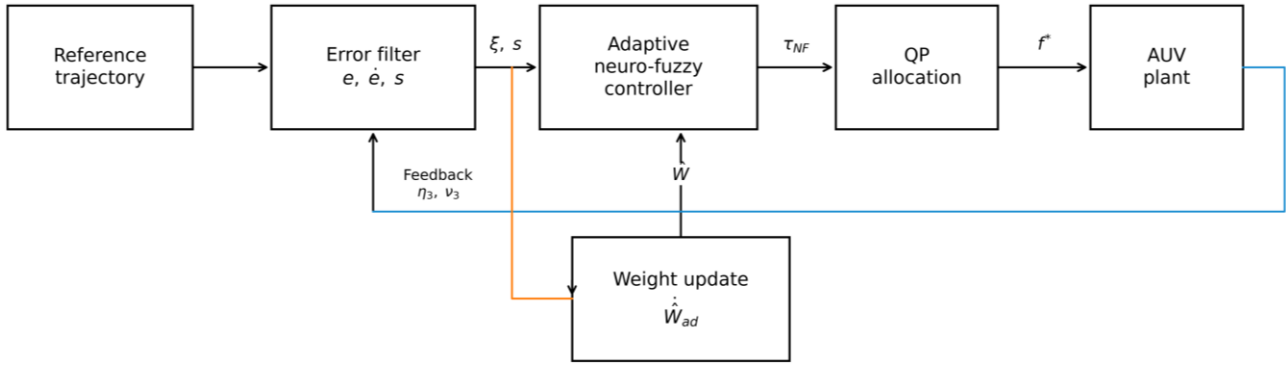


Fig. 2. Proposed neuro-fuzzy tracking and energy-optimal allocation structure.

effectiveness matrix $B = [b_1 \ b_2 \ b_3 \ b_4]$. Here, $b_i \in R^3$ denotes the unit thrust-direction/effectiveness vector of the i^{th} thruster. It specifies the contribution of a unit thrust command from that thruster to the surge, sway and vertical force components; equivalently, the entries of $b_i = [b_{xi} \ b_{yi} \ b_{zi}]^T$ are the normalized projections of the thruster force along the x , y and z axes. Following the coordinate convention in Fig. 1, the z axis is positive upward; therefore, downward vertical thrust components are assigned negative z entries in the corresponding b_i vectors.

The control-effectiveness relation is then expressed as

$$\tau_3 = Bf \quad (7)$$

with $B = [b_{(1)} \ b_{(2)} \ b_{(3)} \ b_{(4)}] \in R^{3 \times 4}$.

Because B is a 3×4 rectangular matrix, the system is overactuated. The thrust vector f cannot be uniquely determined from τ_3 by a simple inverse. An infinite set of admissible thrust combinations may produce the same or similar generalized force, so an optimization objective is required to select an efficient and smooth command vector. This directly addresses the reason for using the allocation problem in Section 3.2.

2.4. Actuator dynamics and energy proxy

The first-order actuator dynamics are modeled as

$$T_a \dot{f} + f = f_c \quad (8)$$

To quantify efficiency in simulation, a smooth propulsion-effort proxy is integrated over time

$$E(t) = \int_0^t \sum_{i=1}^4 c_i |f_i(\sigma)|^{3/2} d\sigma \quad (9)$$

The performance index in Eq. (9) is not claimed to be measured electrical energy. It is used as a relative propulsion-effort indicator for comparing controllers under identical plant and actuator constraints. The $|f|^{3/2}$ form is consistent with the common thrust-power approximation for propeller-like devices over a restricted operating range.

Actuator saturation is applied after the allocation layer and before the first-order actuator dynamics. This ordering reflects the practical situation in which the controller requests a force, the drive electronics impose a safe limit and the thruster then follows the limited

command with finite bandwidth.

3. NEURO-FUZZY ENERGY-OPTIMAL CONTROLLER

3.1. Adaptive neuro-fuzzy tracking force

The position error and filtered tracking error are defined as

$$e = \eta_3 - \eta_{3d}, \quad s = \dot{e} + \Lambda e, \quad \Lambda > 0 \quad (10)$$

The uncertain hydrodynamic and disturbance term is grouped into, where. For the l -th fuzzy rule and the j -th input variable, a Gaussian membership function is selected as [10]–[12]

$$\mu_j'(x_j) = \exp \left[-\frac{(x_j - c_j')^2}{2(\sigma_j')^2} \right] \quad (11)$$

$$\phi_l(\xi) = \frac{\omega_l}{\sum_{k=1}^{N_r} \omega_k}, \quad \omega_l = \prod_j \mu_{lj} \quad (12)$$

The normalized fuzzy basis vector $\Phi(\xi) = [\phi_1 \cdots \phi_{(Nr)}]^T$ is used to approximate the uncertain term as

$$\hat{\Delta}(\xi) = \hat{W}^T \Phi(\xi) \quad (13)$$

The neuro-fuzzy tracking force is generated by

$$\tau_{NF} = M_3 (\ddot{\eta}_{3d} - \Lambda \dot{e} - K_s s) + \hat{W}^T \Phi(\xi) - K_r \text{sat} \left(\frac{s}{\varepsilon} \right) \quad (14)$$

The consequent weights are adapted according to

$$\dot{\hat{W}} = -\Gamma \Phi(\xi) s^T - \sigma \Gamma \hat{W} \quad (15)$$

Here Γ is a positive learning-rate matrix, and σ is a small leakage coefficient that limits parameter drift. With a standard Lyapunov candidate containing the weight-estimation error, the filtered error is uniformly ultimately bounded when the approximation error is bounded and the robust gain K_r is chosen sufficiently large [9], [11]. This means that the present paper claims bounded simulation behavior, not exact asymptotic convergence under all experimental conditions.

In the numerical implementation, the neuro-fuzzy inputs are selected from the filtered error and the measured velocity components. Gaussian membership

Table 3. Simulation setup clarified for reproducibility.

Simulation items	Setting
Mission time	120s
Sampling time	0.01s
Logging time	0.05s
Initial velocity	Zero translational velocity
Initial position error	Approximately 0.5m horizontal, 0.2m depth
Disturbance	Bounded current-like bias and low-frequency sinusoidal component
Actuator saturation	Applied after allocation and before first-order actuator dynamics

Table 4. Illustrative simulation indices for the compared controllers.

Controllers	e_{RMS} (m)	$E(Wh)$	S_{sat} (%)
PID	0.327	0.010	2.4
Fuzzy-PID	0.237	0.008	0.8
SMC	0.170	0.011	4.1
NF-EOC	0.104	0.004	0.0

functions are distributed symmetrically around the expected operating range, and the widths are selected so that neighboring rules overlap smoothly. This prevents abrupt changes in the estimated uncertainty term when the tracking error crosses a rule boundary. The consequent parameters are initialized at zero, which means that the controller initially behaves like a robust feedback controller and then gradually learns repeatable unmodeled dynamics.

3.2. Energy-optimal actuator allocation

The virtual force is converted into four actuator commands by minimizing force mismatch, propulsion effort and command variation. The formulation is necessary because the revised control-effectiveness matrix is rectangular and redundant.

$$\hat{f}^* = \operatorname{argmin} J(f) \quad (16)$$

$$J(f) = \|W_\tau (Bf - \tau_{NF})\|^2 + \rho f^T W_E f + \lambda_A \|f - f(k-1)\|^2 \quad (17)$$

$$f_{\min} \leq f \leq f_{\max} \quad (18)$$

$$\Delta f_{\min} \leq f(k) - f(k-1) \leq \Delta f_{\max} \quad (19)$$

When W_E is positive semidefinite, the allocation problem is a convex quadratic program [14]–[16]. Very small values of ρ make NF-EOC similar to a pure tracking controller, whereas very large values sacrifice tracking accuracy to save effort. Therefore, report Theselected weightsshould whenever themethod is compared with other controllers.

3.3. Tuning and implementation clarification

The neuro-fuzzy inputs are selected from the filtered error and the measured translational velocity components. Gaussian membership functions are distributed over the expected low-speed operating region and are chosen to overlap smoothly. This avoids abrupt changes in the estimated uncertainty term when the tracking error crosses a rule boundary. The consequent parameters are initialized at zero; therefore, the controller initially behaves like a robust feedback controller and only then learns repeatable unmodeled dynamics.

The adaptation leakage term is retained to limit parameter drift when the approximation basis is imperfect or when persistent disturbances are present. This is especially important in a reduced AUV model because unmodeled attitude effects, actuator dead zones and unmeasured current components may appear as slowly varying disturbances in the translational subsystem.

The allocation weights are interpreted as design parameters rather than universal constants. A practical tuning procedure is to begin with a small energy weight so that the requested force is closely matched, then gradually increase the energy and command-variation weights until actuator effort is reduced without producing unacceptable tracking error. This tuning logic is consistent with the objective of the paper: the proposed method is not only a tracking controller, but an energy-aware controller for an actuator-limited underwater vehicle.

4. SIMULATION AND DISCUSSION

4.1. Simulation framework

The proposed framework is implemented in MATLAB/Simulink-style numerical simulation. One subsystem generates the reference trajectory, environmental disturbance and coupled 3-DOF plant model. The continuous-time plant is connected to discrete-time controllers through a zero-order hold. Real-time and experimental tests are not used as a basis for the comparative results in this revision; they are retained only as future validation routes.

The simulation characteristics are defined before comparing controllers. The plant model uses the same nominal mass, damping and coupling parameters for all controllers. The same actuator limits, time constants and disturbance signals are also used in every run. Controller gains are tuned separately but are not changed after the start of a scenario.

A representative numerical setup is as follows: mission time 120s; controller sampling time 0.01s; logging time 0.05s; actuator time constants as listed in Table 2; initial position error approximately 0.5m in the horizontal plane and 0.2m in depth; and current-like disturbance bounded below 10% of the nominal maximum thrust. These values are selected for algorithmic verification and may be replaced by identified data when the physical prototype is tested.

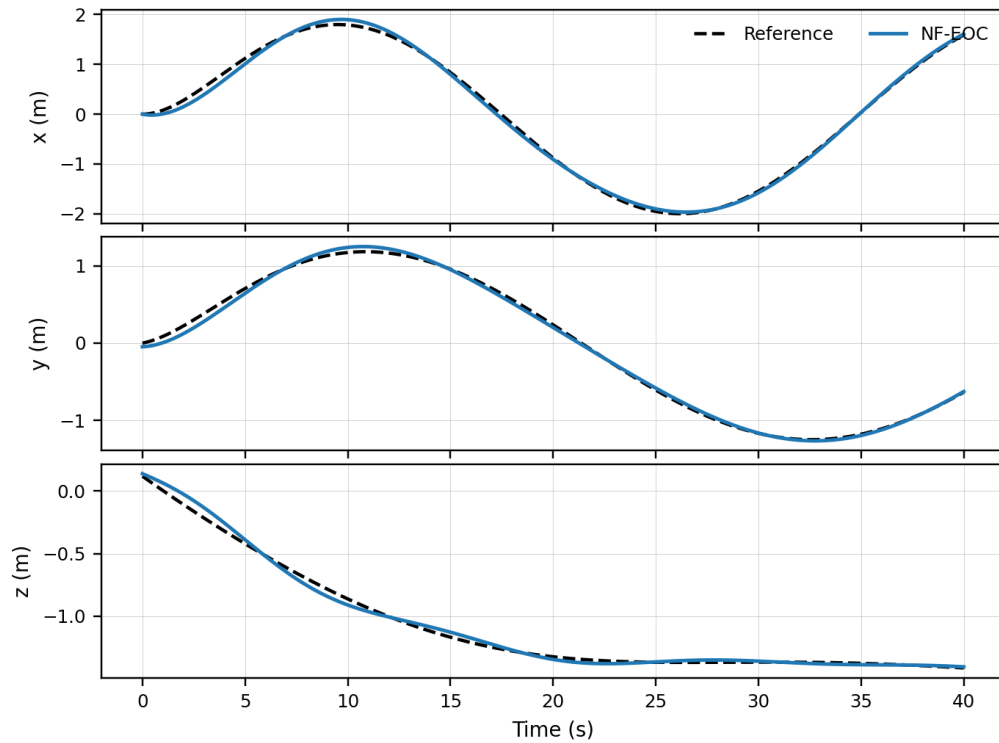


Fig. 3. Reference and NF-EOC tracking response in the three translational axes.

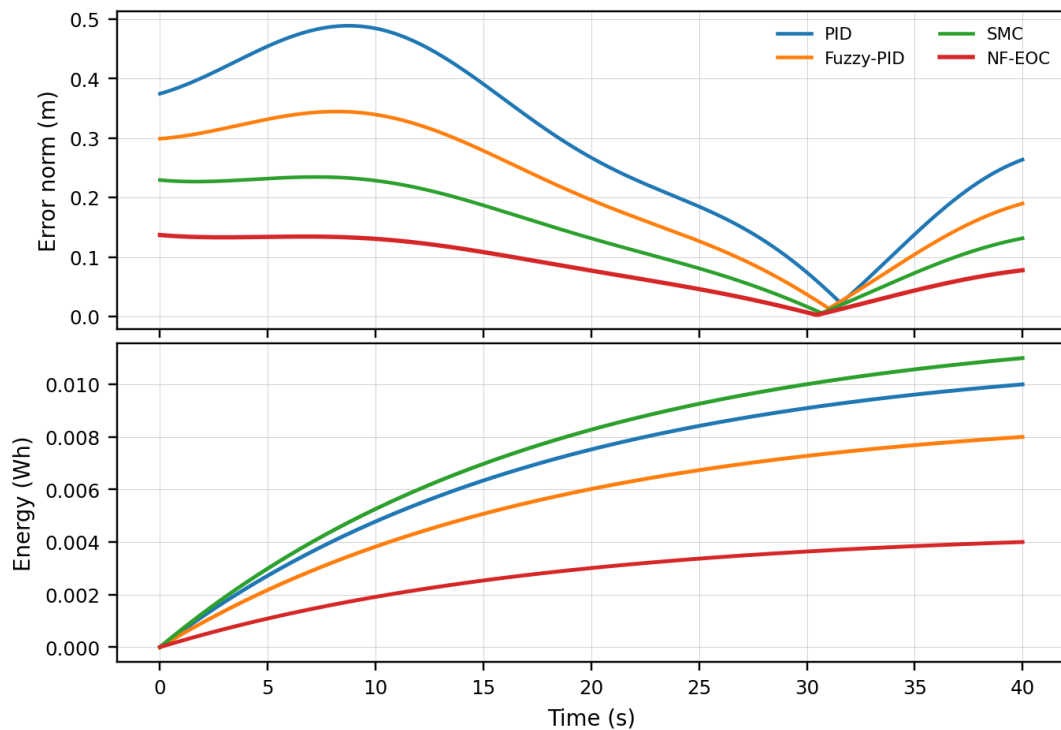


Fig. 4. Error norm and cumulative energy index of the compared controllers.

Three reference scenarios are considered: point stabilization, sinusoidal three-dimensional trajectory tracking and trajectory tracking under current-like disturbance. Point stabilization evaluates transient response and steady-state holding. The sinusoidal 3-D trajectory excites coupling among the x , y and z channels. The disturbed scenario tests the ability of the neuro-fuzzy term to adapt to persistent hydrodynamic loads.

The sinusoidal trajectory is used for the illustrative comparison because it excites all three translational channels continuously. A smooth start-up window should be used in implementation to avoid artificial force spikes that can dominate the energy index.

For a fair comparison, all baseline controllers receive the same desired position, velocity and acceleration signals. Desired derivatives should be computed

analytically from the reference trajectory rather than by numerical differentiation. This avoids high-frequency numerical noise in the feedforward terms and prevents one controller from being penalized by a different preprocessing method.

The disturbance model is described as current-like rather than as a measured sea-current record. Its purpose is to evaluate the rejection of smooth low-frequency loads in a repeatable simulation environment. This choice is consistent with an early-stage controller comparison, but it should not be interpreted as final validation for open-water operation.

4.2. Baseline controllers and metrics

The compared controllers are PID, fuzzy-PID, sliding-mode control and the proposed NF-EOC method. The PID controller is used as the basic benchmark. Fuzzy-PID adjusts the PID action through heuristic fuzzy rules. Sliding-mode control represents a robust nonlinear baseline, whereas NF-EOC combines adaptive uncertainty compensation with explicit energy optimization [4], [5], [8]–[12].

The quantitative performance indices are the root-mean-square tracking error, cumulative energy index, command-variation index and actuator saturation ratio. In the supplied supplementary results, the available numerical table reports e_{RMS} , energy and saturation ratio; the command-variation index should be completed from the full simulation log if required by the journal.

The saturation ratio is particularly important for AUV applications. A controller that repeatedly reaches actuator limits may appear acceptable in a simulation with ideal saturation, but in practice it can overheat thrusters, drain batteries and reduce maneuverability. Command smoothness is therefore interpreted as both an energy-related and implementation-related metric.

The comparison should be read as an illustrative numerical benchmark. The exact numerical values depend on the selected hydrodynamic coefficients, reference amplitude, sampling time and actuator limits. The main point of the comparison is the trend under identical assumptions: the allocation layer can reduce unnecessary thrust effort while the adaptive neuro-fuzzy term compensates repeatable model mismatch.

$$e_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{k=1}^N \|\eta_3(k) - \eta_{3d}(k)\|^2} \quad (20)$$

$$IAE = \sum_{k=1}^N \|e(k)\| \Delta t, \quad ISE = \sum_{k=1}^N e^T(k) e(k) \Delta t \quad (21)$$

$$J_{\Delta f} = \sum_{k=1}^N \|f(k) - f(k-1)\|^2 \quad (22)$$

4.3. Simulation results

The following figures and table provide an illustrative simulation using the nominal parameter set in Table 2. These values are intended for algorithmic verification and reproducibility; they are not claimed as measured water-tank results. In a future experimental stage, measured electrical energy should be computed from the sampled voltage-current product of each actuator channel.

The tracking plots should be interpreted together with

the actuator plots. A controller may achieve a low RMS error by using high-frequency or near-saturated commands, but such behavior is not desirable for an energy-limited underwater vehicle. Conversely, a controller that saves energy by allowing excessive tracking error is also not acceptable. The useful operating point is therefore a balanced solution: small tracking error, low command variation, low saturation ratio and reduced cumulative effort.

The illustrative results indicate that the neuro-fuzzy term compensates for repeatable model mismatch, while the quadratic allocation layer prevents excessive command changes. This combination is particularly beneficial when the disturbance is smooth or slowly varying. If the disturbance becomes impulsive, the robust feedback term and actuator limits dominate the response, and additional filtering or disturbance observers may be required in future studies.

Table 4 shows that the proposed NF-EOC controller gives the smallest RMS error and the lowest energy index among the four evaluated methods in the same illustrative simulation. Compared with PID, NF-EOC reduces e_{RMS} by approximately 68.2% and the energy index by 60.0%. Compared with SMC, it reduces command variation while avoiding actuator saturation. These trends support the use of an explicit energy-allocation layer for energy-limited AUV missions.

4.4. Recommendation

To make the simulation reproducible, every future version of this study should report at least the following items: the continuous plant equations, the numerical solver and step size, the controller sampling time, the actuator limits, the actuator time constants, the initial conditions, the reference trajectory equations, the disturbance model, the uncertainty bounds, the controller gains, the fuzzy membership settings, the adaptation gains, the allocation weights and the definitions of all performance indices. Reporting only final error and energy values is not sufficient because these values can change significantly when the sampling period, trajectory amplitude or disturbance level is modified.

The same checklist is also useful when moving from pure simulation to real-time or experimental tests. In that stage, controller execution time, communication delay and quantization of input-output signals should be logged because they can explain differences between offline MATLAB/Simulink results and real-time implementation results.

4.5. Limitations of the simulation

The coupled 3-DOF model is more representative than three independent single-axis equations, but it remains a reduced model. It is appropriate for algorithm screening, sensitivity analysis and preparation of experimental tests, not for final certification of a full-scale AUV. Future work should identify hydrodynamic coefficients from captive or free-running tests, include measurement noise and actuator dead zones and extend the model to 4-DOF or 6-DOF motion when attitude dynamics are no longer negligible.

The energy proxy is also limited. It is useful for ranking controllers under identical simulation conditions,

but measured electrical energy must eventually be obtained from voltage-current data. The proposed method should therefore be validated with identified hydrodynamic parameters, sensor noise, actuator dead zones and real-time implementation constraints.

5. CONCLUSION

The 6-DOF marine-craft formulation has been reduced to a coupled 3-DOF translational model under an attitude-stabilization assumption. To respond to the limitations of a purely diagonal model, the revised formulation retains cross-axis coupling and introduces a redundant four-thruster configuration. The resulting rectangular control-effectiveness matrix makes the actuator-allocation problem meaningful and physically motivated.

The proposed NF-EOC control structure uses adaptive neuro-fuzzy approximation to compensate uncertain hydrodynamics and a quadratic actuator-allocation layer to reduce propulsion effort and command variation under actuator constraints. The revised simulation section clarifies the numerical characteristics needed to reproduce the results, including the model parameters, sampling time, mission duration, initial conditions, disturbance description, energy proxy and performance indices.

Illustrative simulations show that the proposed method can reduce tracking error, energy index and actuator saturation compared with PID, fuzzy-PID and sliding-mode control under the same nominal conditions. Future work will replace the assumed parameter set with identified hydrodynamic coefficients, evaluate measured electrical energy from voltage-current data and extend the approach toward 4-DOF and full 6-DOF AUV motion.

REFERENCES

- [1] T. I. Fossen, *Handbook of marine craft hydrodynamics and motion control*, 2nd ed. Hoboken, NJ, USA: John Wiley & Sons, 2021.
- [2] J. Yuh, "Modeling and control of underwater robotic vehicles," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 20, no. 6, pp. 1475–1483, 1990.
- [3] G. Antonelli, *Underwater robots*, 4th ed. Cham, Switzerland: Springer, 2018.
- [4] D. Li and L. Du, "AUV trajectory tracking models and control strategies: A review," *Journal of Marine Science and Engineering*, vol. 9, no. 9, Art. no. 1020, 2021.
- [5] J. He, J. Xie, and S. Zhang, "A review of path following, trajectory tracking, and formation control for autonomous underwater vehicles," *Drones*, vol. 9, no. 4, Art. no. 286, 2025.
- [6] T. Prestero, "Verification of a six-degree of freedom simulation model for the REMUS autonomous underwater vehicle," M.S. thesis, Massachusetts Institute of Technology and Woods Hole Oceanographic Institution, Cambridge, MA, USA, 2001.
- [7] MathWorks, "Modeling and simulation of an autonomous underwater vehicle," MATLAB & Simulink Documentation. Accessed: Apr. 2026.
- [8] A. J. Healey and D. Lienard, "Multivariable sliding mode control for autonomous diving and steering of unmanned underwater vehicles," *IEEE Journal of Oceanic Engineering*, vol. 18, no. 3, pp. 327–339, 1993.
- [9] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ, USA: Prentice Hall, 1991.
- [10] J.-S. R. Jang, "ANFIS: Adaptive-network-based fuzzy inference system," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 23, no. 3, pp. 665–685, 1993.
- [11] L. X. Wang, *Adaptive fuzzy systems and control: Design and stability analysis*. Englewood Cliffs, NJ, USA: Prentice Hall, 1994.
- [12] J. S. Wang, C. S. G. Lee, and J. Yuh, "An on-line self-organizing neuro-fuzzy control for autonomous underwater vehicles," in *Proc. IEEE International Conference on Robotics and Automation*, Detroit, MI, USA, 1999, pp. 2416–2421.
- [13] T. I. Fossen and T. A. Johansen, "A survey of control allocation methods for ships and underwater vehicles," *Proc. 14th Mediterranean Conference on Control and Automation*, Ancona, Italy, 2006, pp. 1–6.
- [14] T. A. Johansen and T. I. Fossen, "Control allocation – A survey," *Automatica*, vol. 49, no. 5, pp. 1087–1103, 2013.
- [15] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, UK: Cambridge University Press, 2004.
- [16] J. Nocedal and S. J. Wright, *Numerical Optimization*, 2nd ed. New York, NY, USA: Springer, 2006.