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On the Optimization of Mooring Lines for Floating Structures Considering the Role of Sinker Mass

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Abstract: Conventional mooring line design for floating structures often relies on empirical or trial-and-error methods, leading to inefficient material use and increased costs. This study proposes a multi-objective optimization framework using the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm to design a single mooring line, with a novel emphasis on the sinker's mass. The approach simultaneously minimizes the mooring line weight and sinker mass while satisfying strength, stability, and seabed contact constraints. The resulting Pareto front reveals a clear trade-off between these objectives, with an optimal "knee point" offering a balanced design. Compared to empirical designs, the optimized configuration reduces mooring line weight by 15.9% and sinker mass by 22.3%, demonstrating significant material and cost savings. This method provides a systematic and efficient tool for enhancing the design of mooring systems in offshore engineering applications.

Keywords: Mooring line, floating structure, calculation method, multi-objective particle swarm optimization, sinker, trial and error method.

1. INTRODUCTION

The mooring system of a floating structure is a vital component that ensures the stability of both the floating body and the whole structure. Its primary function is to resist environmental loads such as waves, currents, winds, and external actions from vessels. Depending on the design and operational requirements, the floating body may be moored through a single-point mooring (SPM) or a spread mooring system with multiple lines. The mooring lines may be arranged in either a catenary configuration (slack mooring) or a taut-leg configuration (taut mooring). Typical mooring elements include stud-link or studding chains, wire ropes, and synthetic fiber ropes (e.g., polyester, aramid, HMPE), which are fabricated from high-strength steel alloys or advanced non-metallic materials [1, 2]. Currently, the design of mooring lines primarily relies on iterative trial-and-error methods or the personal experience of designers, which may lead to inefficiencies and material waste. Therefore, it is essential to adopt optimal calculation approaches to enhance design accuracy and resource utilization [1, 3].

Since the early 2000s, optimization of mooring lines has been paid attention to, driven by the emergence and advancement of modern optimization methods. In 2007, Shafieefar *et al.* [4] proposed a procedure for the optimization of the mooring design of floating platforms using GA algorithm. The objective function is to minimize the floating body's offset. However, only the safety factor of mooring lines was considered. Harmony Search was utilized to optimize the mooring cost of FPSO in reference [5]. In this research, only three constraints were applied, including maximum platform offset, factor of safety for intact case top tension, and zero-degree angle of anchor. Besides GA, PSO and improved PSO were used to optimize the platform offset [6-8]. While factors such as floating body offset, mooring line tension, and seabed obstacles were taken into account, the influence of the sinker or mooring anchor was notably omitted from the analysis.

This paper presents the optimization of a single mooring line subjected to a constant horizontal load considering the role of the sinker alongside the floating body offset and line tension. A Pareto font will be obtained to show the relation between the mass of the

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Research article

sinker and the optimal parameter of the mooring line.

2. BASIC EQUATIONS OF A SINGLE MOORING LINE

The theory of mooring line calculation has been developed since the 20th century [3]. Most of the authors proposed catenary equations to describe the shape of the mooring line as follows:

$$z(x) = a\left\{\cosh\left[\frac{x}{a} + \operatorname{arsinh}(\tan\theta_0)\right] - \cosh[\operatorname{arsinh}(\tan\theta_0)]\right\} \quad (1)$$

$$x(l) = a\left[\operatorname{arsinh}\left(\frac{l}{a} + \tan\theta_0\right) - \operatorname{arsinh}(\tan\theta_0)\right] \quad (2)$$

$$a = \frac{H}{w} \quad (3)$$

where, x, z denote the spatial coordinates of the mooring line, θ_0 is the mooring line angle at the seabed as shown in Fig. 1. H corresponds to the total transverse force exerted on the floating body, w is the unit weight of the mooring line in water. Water depth h, floating body offset S, and mooring line length l can be calculated as follows:

$$h = a\left[1 + \left(\frac{l}{a} + \tan\theta_0\right)^2\right]^{\frac{1}{2}} - \frac{a}{\cos\theta_0} \quad (4)$$

$$S = a\left[\operatorname{arsinh}\left(\frac{l}{a} + \tan\theta_0\right) - \operatorname{arsinh}(\tan\theta_0)\right] \quad (5)$$

$$l(x) = a\left\{\sinh\left[\frac{x}{a} + \operatorname{arsinh}(\tan\theta_0)\right] - \tan\theta_0\right\} \quad (6)$$

Also, the tension along the mooring line is calculated as shown in the following equation:

$$T(l) = H\sqrt{1 + \left(\frac{l}{a} + \tan\theta_0\right)^2} \quad (7)$$

3. OPTIMIZATION OF A SINGLE MOORING LINE

3.1. Multi-objective particle swarm optimization (MOPSO) algorithm

Multi-objective particle swarm optimization (MOPSO) has been proposed by Moore *et al.* in some publications from 1999 to 2000 and developed in the works of Coello and Víctor Martínez-Cagigal [9-11]. In this approach, the concept of Pareto dominance is integrated into the standard PSO framework, enabling the algorithm to effectively address optimization problems involving multiple conflicting objectives. Each particle in the swarm is guided by both its own experience and the information retrieved from the archive, enhancing the algorithm’s ability to explore and exploit the solution space effectively. To ensure diversity among solutions and avoid premature convergence, MOPSO integrates various diversity-preservation strategies, such as solution dispersion mechanisms and random selection from the archive. These techniques help maintain a well-distributed set of solutions across the Pareto front.

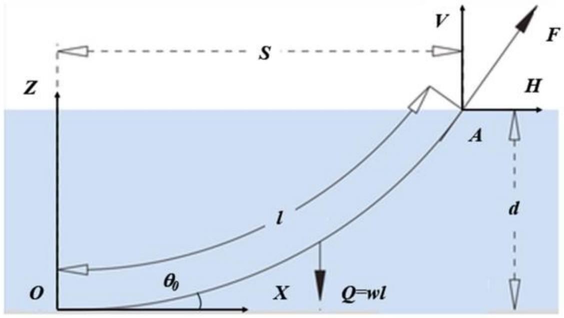

Fig. 1. Diagram of a single mooring line [12].

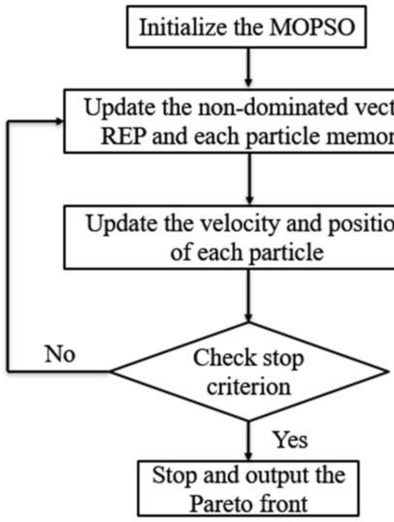

Fig. 2. PSO flowchart.

Fig. 2 illustrates the flowchart of MOPSO algorithm. The process begins with the initialization of particles and parameters. In each iteration, particles update their velocities and positions according to their personal and global best solutions. The repository of non-dominated solutions (REP) is continuously updated to maintain the Pareto-optimal set. The process repeats until the stopping criterion is satisfied, after which the final Pareto front is obtained.

The MOPSO algorithm is selected for this study due to its simplicity, efficiency, and strong capability in handling multi-objective optimization problems. Unlike traditional evolutionary algorithms such as NSGA-II or MOEA/D, MOPSO requires fewer control parameters and achieves faster convergence while maintaining good solution diversity. Moreover, MOPSO has been widely applied and proven effective in various engineering fields, including offshore engineering, where it demonstrates high reliability in optimizing mooring system configurations under multiple conflicting design criteria.

3.2. Optimization of single mooring line

This section presents the optimization of a single mooring line of a buoy. As the scope of this research is the mooring line, the buoy is assumed to be stable under the environmental loads. Consequently, the floating body maintains equilibrium, avoiding both submergence and uplift from the water surface. Therefore, the vertical force transmitted from the buoy to the mooring line can be considered negligible.

To study the role of the sinker's mass, the total environmental load acting on the floating body is assumed to be horizontal and constant of $H = 3.37\,T$. The water depth is $h = 50m$. The values above are referred to the reference [13]. The optimal results will be compared with the mooring line and sinker's parameters from this document.

MOPSO is utilized with two objective functions as follows:

$$f_1 = wl \quad (8)$$

$$f_2 = m_s \quad (9)$$

where, m_s is the mass of the sinker in water. The variables and their boundaries are listed in the following table.

The constraints are listed as follows:

$$S > 0 \quad (10)$$

$$T(l) < [F] \quad (12)$$

$$S^2 > l^2 + h^2 \quad (13)$$

$$T(0)\cos\theta_0 < [m_s - T(0)\sin\theta_0]k_f \quad (14)$$

$$T(0)\sin\theta_0 < m_s/s_f \quad (15)$$

where, k_f denotes the friction coefficient between the sinker and the seabed, while s_f is the safety factor. Equation (11) defines the tensile strength condition of the mooring line, and Equation (13) corresponds to the sliding stability criterion of the sinker. Finally, Equation (14) ensures that the sinker remains in contact with the seabed, preventing any uplift due to vertical forces. The optimization calculation is performed with the parameters shown in Table 2.

It should be noted that the equations (13) and (14) provide the essential relationship to determine the sinker's mass based on the tension of the mooring line at the seabed. The sinker maintains equilibrium through the combined effects of its self-weight and the frictional resistance at the seabed interface. The mass of the sinker counteracts the vertical component of the tensile force transmitted by the mooring line, while the friction between the sinker and the seabed stabilizes it against sliding induced by the horizontal component of the line tension. In this case, the friction coefficient between the sand and the concrete surface is assumed to be 0.6 [13].

Table 1. Variables and boundaries.

No	Variables	Boundaries
1	θ_0(rad)	0 ÷ 0.785
2	l(m)	50 ÷ 150
3	q(kg/m)	5 ÷ 100
4	m_s(kg)	0 ÷ 20000

Table 2. Parameters of MOPSO model.

No	Parmeters	Values
1	Number of particles (N_p)	1000
2	Repository size (N_r)	600
3	Maximum generations (iterations)	600

Table 3. Optimal solution.

Parameters	Optimal solutions	Empirical design
θ_0(rad)	0.519	0.448
l(m)	78.2	81.0
q(kg/m)	24.09	27.45
m_s (kg)	5625	7237
of_1(kg)	1869	2223
of_2(kg)	5625	7237

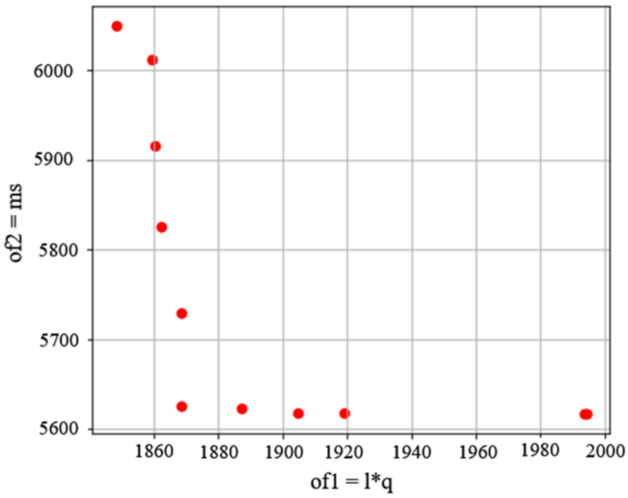

Fig. 3. Pareto front.

3.3. Results and discussions

Fig. 3 presents the Pareto front obtained from the multi-objective optimization using the MOPSO algorithm, where the horizontal axis represents the total weight of the mooring line and the vertical axis represents the sinker mass. The curve clearly illustrates the compromise between these two design objectives. In general, a heavier sinker allows for a lighter mooring line, contributing to improved system stability and reduced tension. However, this relationship is not linear and exhibits diminishing returns beyond a certain point.

It should be noted that while the mooring line weight increases slightly from 1858 kg to 1869 kg, the sinker mass drops significantly from 6011 kg to 5625 kg. This indicates that a small increase in mooring line weight can lead to a substantial reduction in sinker mass, which may be beneficial in terms of cost and installation effort. However, as the mooring line weight continues to increase from 1869 kg to 1994 kg, the sinker mass only decreases marginally from 5625 kg to 5617 kg. This shift marks the onset of diminishing returns, where further investment in heavier mooring lines yields minimal benefit in reducing sinker mass.

The behaviour above is typical in multi-objective optimization problems, where improving one objective beyond a certain threshold leads to small gains in the other. The Pareto front helps designers identify an optimal balance point—where both the mooring line and sinker are sized efficiently without unnecessary material usage or cost escalation. Selecting a solution near the knee point of the Pareto front offers the best trade-off between the mass of the sinker and the mass of the mooring line.

Table 3 presents the optimal solution where the value of the two objective functions is selected at the knee point of the Pareto front. The solution is also compared to the empirical design parameters in reference [13]. Results show that the total mass of the sinker and the mooring line can be reduced by 15.9% and 22.3% respectively, by using optimal parameters.

4. CONCLUSIONS

This study developed a multi-objective optimization framework for designing a single mooring line of a floating buoy, with a novel focus on the role of the sinker's mass. Utilizing the MOPSO algorithm, the proposed approach generated a well-distributed Pareto front, revealing the trade-off between mooring line weight and sinker mass. From the Pareto set, designers should select the higher and closest values of the sinker mass and the submerged unit weight of the mooring line based on relevant design standards and manufacturer data to ensure practical feasibility.

The analysis identified a "knee point" on the Pareto front, representing an optimal balance that minimizes material use while ensuring structural reliability. Compared to empirical design parameters, the optimized solution reduced the mooring line weight by 15.9% and the sinker mass by 22.3%, demonstrating significant material and cost savings. These findings highlight the potential of MOPSO as a robust tool for enhancing the efficiency and sustainability of mooring system designs. Future research could extend this approach to 3D problem with multi-line mooring systems and incorporate dynamic environmental conditions, such as wave and current variations, to further improve its applicability to complex offshore environment.

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