

A Real-World Application of Multi-Optimization for a Berthing Structure Using the MOMVO Algorithm

Truong Vu Huu* 

Abstract: The Multi-Objective Multi-Verse Optimizer (MOMVO) is applied to the structural optimization of a real-world berthing dolphin (BD). The BD structure is represented in SAP2000 using a detailed finite element model of piles and cap, with berthing and mooring loads defined in accordance with international guidelines. Two conflicting objectives are considered simultaneously: minimizing construction cost and minimizing the maximum structural displacement. Structural and geotechnical requirements are incorporated through a penalty-based constraint handling scheme to ensure compliance with Vietnamese design standards. The optimization results indicate that MOMVO generates well-distributed Pareto-optimal solutions that outperform the existing design. Relative to the baseline configuration, the optimal solutions achieve cost reductions of up to 70% while significantly decreasing structural displacement, with several compromise solutions available to balance both objectives. The results demonstrate that the proposed MOMVO–FEM framework provides practical and feasible design alternatives for improving the efficiency and safety of pile-supported marine structures.

Keywords: Berthing dolphin, Finite element analysis, Multi-objective optimization, Multi-objective multi-verse optimizer, SAP2000.

1. INTRODUCTION

Designing marine infrastructure, such as berthing dolphins (BDs), involves balancing several conflicting requirements, most notably structural safety and construction costs. Conventional design approaches are commonly based on deterministic procedures and prescriptive code checks. Although such approaches provide reliable and standardized solutions, they offer limited flexibility in exploring a broad design space or in explicitly identifying trade-offs among competing performance criteria. In berthing structures, pile foundations must resist significant axial and lateral loads induced by berthing and mooring actions, while excessive material use and construction cost should be avoided. These conflicting requirements naturally frame BD design as a multi-objective optimization problem.

Multi-objective optimization (MOO) provides a systematic framework for addressing engineering problems with competing objectives. Unlike single-objective formulations, MOO produces a set of Pareto-optimal solutions that represent different trade-offs among design criteria. This characteristic allows designers to select solutions that best align with

functional, economic, or safety priorities. Population-based metaheuristic algorithms, such as Genetic Algorithms [1], Particle Swarm Optimization [2], and Ant Colony Optimization [3], are widely adopted in MOO due to their ability to handle nonlinear behavior, complex constraints, and high-dimensional search spaces [4]. Established approaches, including NSGA-II [5], MOPSO [6], and MOEA/D [7] emphasize both convergence toward the Pareto front and the preservation of solution diversity.

The Multi-Verse Optimizer (MVO) is a physics-inspired metaheuristic that models search behavior using concepts such as white holes, black holes, and wormholes [4]. Its multi-objective extension, MOMVO, incorporates an external archive and a leader selection strategy to guide the search toward well-distributed Pareto-optimal solutions [4]. Previous methodological and benchmark studies demonstrate that MOMVO exhibits a balanced exploration–exploitation behavior and maintains competitive performance in complex optimization landscapes. These characteristics are particularly relevant for engineering applications that require strict structural and geotechnical constraints to be satisfied.

Manuscript received October 30, 2025; received in revised form December 13, 2025; accepted December 24, 2025; available online March 30, 2026.

Truong Vu Huu is with the Faculty of Civil Engineering, Vietnam Maritime University, Haiphong, Vietnam (e-mail: truongvh.ctt@vimaru.edu.vn).

* Corresponding author.

In practical structural optimization problems, especially those involving detailed finite element models, the computational cost of evaluating the objective function becomes a critical factor. Each candidate design often requires a complete structural analysis, which limits the feasibility of extensive algorithmic benchmarking. Under such conditions, the suitability of an optimization algorithm is closely related to its robustness and its ability to identify high-quality solutions within a limited number of evaluations. The present work, therefore, emphasizes the application of MOMVO to a real-world berthing dolphin structure, focusing on its integration with a commercial finite element platform rather than on comparative algorithmic performance.

A detailed finite element model of the berthing dolphin is developed in SAP2000 to evaluate structural responses under combined berthing and mooring loads in accordance with international recommendations. Two conflicting objectives are considered simultaneously: minimizing construction cost and minimizing maximum structural displacement. Structural and geotechnical constraints are enforced in accordance with Vietnamese design standards. Optimization is carried out for various population-size settings, and the resulting Pareto-optimal solutions are evaluated in relation to the existing design. The results demonstrate that the MOMVO–FEM framework is capable of generating feasible and practical design alternatives that enhance structural performance while reducing construction costs, thereby supporting informed decision-making in marine infrastructure design.

2. MULTI-OBJECTIVE MULTI-VERSE OPTIMIZER

2.1. Multi-verse optimizer

The Multi-Verse Optimizer (MVO) originates from the multi-verse theory in cosmology, which introduces three key mechanisms: white holes, black holes, and wormholes [4]. These mechanisms are modeled as exploration, exploitation, and local search operators. Each candidate solution is represented as a universe, each decision variable is treated as an object within that universe, and the fitness of a solution is expressed as its inflation rate.

The search process is divided into two leading operators [4]:

1. White hole–black hole tunnels (exploration):

Universes with higher inflation rates (better fitness) are more likely to share variables, while weaker universes receive them. The exchange follows a roulette wheel mechanism based on the normalized inflation rate

$$x_{i,j} = \begin{cases} x_{k,j}, & r_1 < NI(U_i) \\ x_{i,j}, & r_1 \geq NI(U_i) \end{cases} \quad (1)$$

where $x_{i,j}$ is the j^{th} variable of the i^{th} universe, $NI(U_i)$ is its normalized inflation rate, $x_{k,j}$ is the variable from a selected universe, and $r_1 \in [0,1]$ is random.

2. Wormholes (exploitation):

Regardless of the inflation rate, universes may shift toward the best solution obtained so far. This local search

Pseudo-code 1. Simplified implementation of the proposed MOMVO–FEM (SAP2000) optimization framework for BD design.

```

Initialize the population of universes  $X_i (i = 1, \dots, N)$  within bounds
Set iteration  $l = 1$  and maximum iteration  $L$ 
While  $l \leq L$ 
    Evaluate objectives  $f_1(X_i), f_2(X_i)$  using FEM analysis (SAP2000)
    Apply non-dominated sorting  $\rightarrow$  obtain Pareto fronts
    Update inflation rate = rank-based fitness of universes
    For each universe  $i$ 
        # White/black hole exchange (exploration)
        Select universe  $k$  by the roulette wheel based on the inflation rate
        Replace the random variables of  $X_i$  by  $X_k$ 
    if  $\text{rand} < NI(X_i)$ 
        # Wormhole update (exploitation)
        If  $\text{rand} < WEP$ 
             $X_i = \text{Best\_universe} \pm \text{TDR} \times \text{rand} \times (u_b - l_b)$ 
        End if
    End for
    Update  $WEP$  and  $TDR$  adaptively
    Save non-dominated solutions to the Pareto archive
     $l = l + 1$ 
End while
Return the final Pareto front of cost-displacement trade-offs

```

mechanism is formulated as

$$x_{i,j} = \begin{cases} X_j + TDR((ub_j - lb_j)r_4 + lb_j), & r_3 < 0.5, r_2 < WEP \\ X_j - TDR((ub_j - lb_j)r_4 + lb_j), & r_3 \geq 0.5, r_2 < WEP \\ x_i^j, & r_2 \geq WEP \end{cases} \quad (2)$$

where X_j is the best universe variable, ub_j and lb_j are bounds, WEP is the wormhole existence probability, TDR is the traveling distance rate, x_i^j indicates the j^{th} parameter of i^{th} universe, and $r_2, r_3, r_4 \in [0,1]$.

The parameters adapt across iterations

$$WEP = WEP_{min} + \frac{l}{L}(WEP_{max} - WEP_{min}) \quad (3)$$

and

$$TDR = 1 - \left(\frac{l^{1/p}}{L^{1/p}}\right) \quad (4)$$

where l is the current iteration, L the maximum iterations, and p controls exploitation accuracy.

2.2. Extension to multi-objective

For multiple objectives, an external archive maintains non-dominated solutions. The update mechanisms remain the same, but leader solutions are drawn from the archive using a probability model based on crowding distance. The selection probability of an archive member i according to [8] is

$$P_i = \frac{1}{(n_i)^c} \quad (5)$$

where n_i is the local density around solution i , and $c > 1$ is a constant. This mechanism encourages solutions

Table 1. Load cases and definitions.

| Load case | Type | Design type | Values | | | References |
|-----------|--------|-------------|--------|--------|--------|--------------------------------|
| | | | X (kN) | Y (kN) | Z (kN) | |
| DL | Static | Dead | --- | --- | --- | Computed internally by SAP2000 |
| BL | Static | Live | 222.42 | 444.83 | 0 | [11, 12] |
| ML | Static | Live | 99.05 | 118.07 | 71.2 | [11, 12] |
| LL | Static | Live | 0 | 0 | 9.81 | [11, 12] |

Table 2. Load combinations for BD.

| Load cases | Combinations (Linear add) | | |
|-------------------------------|---------------------------|-------|-------|
| | COMB1 | COMB2 | COMB3 |
| DL | X | X | X |
| BL | --- | X | --- |
| ML | --- | --- | X |
| Envelop-Combination (EV-COMB) | X | X | X |



Fig. 1. Berthing structure (red box) for LNG terminal (Source: Google Map).

from sparse regions of the Pareto front.

When the archive is complete, crowded solutions are removed with inverse probability [8]

$$P_i = (n_i)^c \quad (6)$$

Thus, MOMVO balances convergence (guided by wormholes toward leaders) and diversity (maintained by density-based archive pruning).

Building upon this foundation, the present study develops an MOMVO framework incorporating a non-dominated sorting scheme and couples the optimization process with FE evaluations of structural performance. Within each iteration, the algorithm exchanges information among universes according to their inflation rates and updates design variables while retrieving structural responses (cost and displacement) from the FEM analysis (SAP2000). The proposed optimization procedure is implemented through an iterative coupling between MATLAB and SAP2000. In each iteration, MOMVO generates candidate pile designs in MATLAB, which are subsequently analyzed in SAP2000 to obtain the corresponding structural responses, including construction cost and maximum displacement. These

results are then returned to MATLAB for objective evaluation, constraint checking, and population update. The main computational steps of this process are summarized in Pseudo-code 1.

3. MULTI-OPTIMIZATION FOR A BERTHING STRUCTURE

3.1. Berthing structure and FEM model

BDs represent robust, pile-supported marine structures that incorporate high-capacity fenders to absorb and redistribute the kinetic energy and horizontal forces generated during berthing operations. They also provide mooring support through the use of dolphins, thereby improving the safety and functionality of the jetty system (See Fig. 1).

The BD's current design consists of 19 prestressed concrete piles (D600B), each with a length of 39 m, arranged to deliver multidirectional resistance against berthing loads. The layout includes 09 vertical piles dedicated to direct axial load transfer, 06 in-plane inclined piles with a 6:1 slope, and 04 spatially inclined piles at a 6:1 slope but rotated by 30° or 60° to

Table 3. Variables of BD.

| Variable | Description | Type | Range |
|----------|---|----------|------------|
| X_1 | Prestressed-pile outside diameter (mm) | Discrete | Ref. [10] |
| X_2 | Prestressed-pile wall thickness (mm) | Discrete | Ref. [10] |
| X_3 | Inclination angle relative to the z -axis ($1:X_3$) | Discrete | [6,7,8] |
| X_4 | Pile length (m) | Discrete | [1:0.1:40] |

Table 4. Soil properties.

| Layer no. | Soil type | State | Thickness (m) | Consistency index (I_B) |
|-----------|-----------|-------------------|---------------|-----------------------------|
| 1. | Clay | Soft to very soft | 4.8 | 0.76 |
| 2. | Clay | Stiff to hard | 5.3 | 0.31 |
| 3. | Clay | Soft | 9.6 | 0.63 |
| 4. | Clay | Stiff to hard | 1.7 | 0.35 |
| 5. | Clay | Soft | 4.9 | 0.67 |
| 6. | Sand | Medium-grained | 2.3 | 0.20 |

Table 5. Optimal results of the BD structure compared with the current design.

| | Case 1 | | | Case 2 | | | Current design |
|--------------|----------|----------|----------|----------|----------|----------|----------------|
| | A | B | C | I | II | III | |
| Pile type | 800A | 1000A | 1200C | 700A | 1000A | 1200AB | 600C |
| Price (\$/m) | 40.42 | 63.88 | 163.16 | 32.16 | 63.88 | 132.05 | 42.99 |
| X_1 (m) | 0.8 | 1 | 1.2 | 0.7 | 1 | 1.2 | 0.60 |
| X_2 (m) | 0.11 | 0.13 | 0.15 | 0.1 | 0.13 | 0.15 | 0.10 |
| X_3 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| X_4 (m) | 16 | 16.1 | 16.8 | 16.9 | 16.1 | 16.8 | 39.00 |
| f_1 (\$) | 12086.85 | 19221.93 | 51263.73 | 10164.82 | 19221.93 | 41491.17 | 31640.22 |
| f_2 (m) | 0.013 | 0.006 | 0.003 | 0.019 | 0.006 | 0.003 | 0.0333 |

strengthen resistance against horizontal loads.

Above the pile foundation, a cast-in-place cap of grade C40/50 concrete, with dimensions of $8.4 \times 9.6 \times (2.0-3.0)$ m, is constructed. This cap incorporates a reinforced concrete extension beneath the fender installation area, enhancing its flexural and shear resistance to berthing impacts. It also expands the effective anchoring surface for high-capacity fenders. A finite element model (FEM) of the BD structure is established in SAP2000 to assess its structural behavior under practical loading conditions. The model represents the whole geometry of both piles and the cap. Dynamic effects from berthing loads (BL) and mooring loads (ML) are incorporated according to the standard load combinations recommended in PIANC (2002) [9] and OCDI (2002) [10], as shown in Tables 1 and 2.

The governing envelope combination (EV-COMB) accounts for the most critical response scenarios, ensuring a reliable evaluation of structural performance. SAP2000 provides maximum displacements, internal forces, and pile-foundation moments from this analysis, which are subsequently integrated into the MATLAB-based optimization procedure. The interaction between MATLAB and SAP2000 is implemented via the

SAP2000 Open Application Programming Interface (OAPI), which allows automatic updating of pile geometry, execution of finite element analyses, and extraction of structural responses during each optimization iteration.

3.2. Multi-objective functions

The multi-objective nature involves the simultaneous consideration of conflicting design criteria. This study addresses two primary objectives: (1) minimizing the overall construction cost by reducing material consumption and pile dimensions, and (2) maximizing structural performance by limiting displacements and internal forces under critical loading scenarios. These objectives are evaluated within the FEM - optimization framework, where SAP2000 provides accurate structural responses and MOMVO searches for optimal trade-offs among the design variables. The resulting Pareto front offers a diverse set of solutions, enabling informed decision-making in balancing cost efficiency and structural reliability for BD design as

$$\text{minimize}_X F(X) = \{f_1(X), f_2(X)\} \quad (7)$$

where $f_1(X)$ is the construction cost and is computed as

$$f_1(X) = \sum_{i=1}^{N_p} L_i \times P_p \quad (8)$$

where N_p denotes the total number of piles, $L_i \equiv X_4$ is the length of the i^{th} pile, and P_c is the unit price per meter of pile, which depends on the pile type defined by its outer diameter X_1 and wall thickness X_2 . The construction cost objective is therefore explicitly governed by the pile geometry variables: the pile diameter and wall thickness determine the pile type and corresponding unit cost, while the pile length directly controls the total material quantity. The pile inclination variable X_3 influences the global stiffness and load-transfer mechanism of the foundation system, and thus affects the feasible design space, although it does not explicitly enter the cost function. Since functional and construction requirements predetermine the pile cap dimensions, its cost remains constant and is excluded from the optimization process, making $f_1(X)$ an effective measure of the overall construction cost associated with the pile foundation system. In addition, $f_2(X)$ represents the maximum displacement of the BD structure obtained from the FEM analysis in SAP2000 (See Fig. 2). The vector X collects all design variables considered in this case study, as summarized in Table 3.

3.3. Constraint conditions

A penalty-based constraint handling technique guarantees that all candidate designs satisfy geotechnical and structural safety requirements. The constraints assess the axial bearing resistance and bending capacity of the piles, as determined from SAP2000 simulations and verified against the provisions of the geotechnical standard TCVN 10304:2014 [9] as

$$g_1(X) = \frac{R_{c-FEA}}{N_{c,d}} \leq 1 \quad (9)$$

$$g_2(X) = \frac{M_{2,3-FEA}}{M_{c,r}} \leq 1 \quad (10)$$

where R_{c-FEA} represents the total axial force acting on the pile obtained from the SAP2000 analysis, and $M_{2,3-FEA}$ denotes the maximum bending moments about the principal axes. The parameter $M_{c,r}$ is the ultimate cracking moment capacity of the pile section specified in TCVN 7888:2014 [10], and $N_{c,d}$ is the axial bearing resistance determined under site-specific geotechnical conditions following TCVN 10304:2014 [9]. These inequality constraints ensure that the axial load demand does not exceed the allowable bearing capacity and that the bending response remains within the structural resistance of the pile section

$$N_{c,d} = \frac{\gamma_0}{\gamma_n \gamma_k} R_{c,u} = \gamma_c (u \sum \gamma_{cf} f_i l_i + \gamma_{cq} q_b A_b) \quad (11)$$

in which γ_0 denotes the working condition factor that accounts for the improved uniformity of soil with pile foundations, γ_n represents the structural importance factor of the construction, and γ_k is the reliability coefficient of soils. The term q_b corresponds to the unit end-bearing resistance beneath the pile tip, modified by the condition factor γ_{cq} reflecting installation technique and soil state, and multiplied by the effective tip area A_b . The second component of the equation represents shaft resistance, where u is the pile perimeter, γ_{cf} is the working condition factor of the soil along the pile shaft, f_i is the average unit friction resistance of the i^{th} soil layer, and l_i is the embedded length of the pile within

that layer. This formulation integrates both tip and shaft resistance to capture the full axial capacity of the pile in accordance with TCVN 10304:2014.

Additionally, a further geotechnical constraint requires that the pile tip be fully embedded in a competent bearing stratum to ensure axial and lateral stability. This layer must satisfy two conditions: (1) the soil type is classified as dense sand, stiff clay, or rock; and (2) in the case of cohesive soils, the consistency index I_B (plasticity index ratio) must meet the minimum threshold defined by design standards as

$$g_3(X) = I_B - 0.35 \leq 0 \quad (12)$$

Furthermore, the soil layer at the pile tip must satisfy an additional thickness requirement to ensure adequate bearing performance. Specifically, the competent stratum must present a minimum thickness h_{tip} of 2 m, which provides sufficient vertical loading resistance and reduces excessive settlement under service conditions. This criterion complements the requirements on soil type and consistency index, thereby guaranteeing that the pile foundation develops both axial and lateral stability in accordance with geotechnical design principles

$$g_4(X) = h_{tip} - 2 \geq 0 \quad (13)$$

It is noted that the pile length variable represents the total pile length, including an approximately 10 m unembedded segment above the ground surface; therefore, all geotechnical constraints are evaluated based on the effective embedded length of the pile.

3.4. Penalty method

A penalty strategy is applied to guarantee that all design solutions remain consistent with geotechnical capacity and structural safety requirements. In this approach, the objective functions are modified by introducing a penalty term whenever any constraint is violated. The modified multi-objective formulation is expressed as

$$\text{minimize}_X \hat{F}(X) = \{f_1(X) + P(X), f_2(X) + P(X)\} \quad (14)$$

where $P(X)$ denotes the penalty function defined as

$$P(X) = \sum_{i=1}^m \alpha_i \times \max(0, g_i(X)) \quad (15)$$

with $g_i(X)$ representing the constraint functions, α_i the corresponding penalty coefficients, and m the total number of constraints. This mechanism penalizes infeasible designs in proportion to the degree of violation, guiding the search process toward feasible regions of the design space. The penalty values can be dynamically adjusted during the iterations to maintain a balance between global exploration and strict enforcement of feasibility. This integration ensures that the flexibility of the metaheuristic search is preserved while maintaining compliance with practical engineering standards.

3.5. Soil properties

The soil conditions at the project site reveal a predominantly cohesive soil profile, interspersed with sandy deposits at deeper depths. As detailed in Table 4, the soil stratigraphy consists of six layers, primarily composed of soft to stiff clay with varying degrees of consistency. The uppermost layer includes very soft clay with a high consistency index ($I_B = 0.76$), while deeper layers exhibit more consolidated characteristics. Notably, the sixth layer transitions to a medium-grained sand

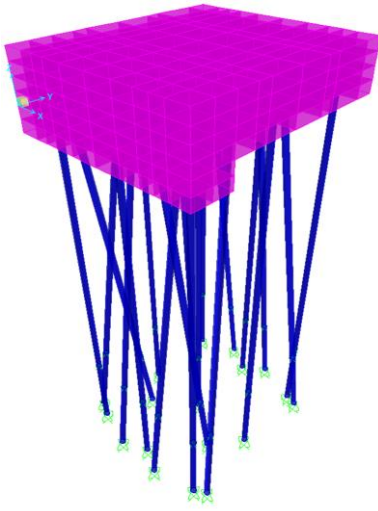


Fig. 2. SAP2000 model for optimization process.

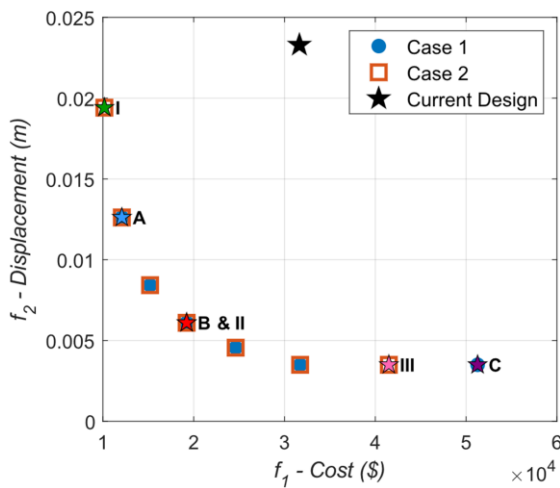


Fig. 3. The Pareto front result of the BD structure obtained by MOMVO was compared with the current design.

deposit with a relatively low consistency index ($I_B = 0.20$). These variations in soil type and strength parameters are critical for evaluating foundation behavior and pile design.

4. RESULTS AND DISCUSSIONS

For the optimization of the BD structure, the initial parameters for the MOMVO algorithm are integrated, including a maximum iteration number of 300, a repository with 100 slots to store non-dominated solutions, and a fixed grid size of 10 to control the spread of solutions. In addition, the population size is set in two cases: Case 1: $N_{pop}=100$ and Case 2: $N_{pop}=200$.

The optimization results obtained from the MOMVO algorithm under two scenarios are presented in the Pareto front in Fig. 3 and summarized in the representative solutions table (Table 5). The Pareto front describes the trade-off between construction cost f_1 and maximum displacement f_2 of the BD structure. Selected solutions (A, B, C for Case 1 and I, II, III for Case 2) represent

typical optimal solutions on the Pareto boundary and are compared with the current design.

Fig. 3 illustrates that MOMVO identifies several solutions that are clearly superior to the current design. The "Current Design" point is located above and to the right of the Pareto front, meaning it has a higher cost and larger displacement. In contrast, the Pareto solutions spread from the low-cost region to the low-displacement region, effectively showing that the algorithm balances global search and local refinement. Increasing the population from 100 to 200 agents only extends the low-cost region (solution I), while the overall front remains essentially unchanged. It demonstrates that MOMVO produces stable results regardless of population size.

To illustrate the trade-offs more clearly, Table 5 lists the selected Pareto solutions from both cases alongside the current design, providing a basis for quantitative evaluation of cost savings and performance improvements.

- Low-cost group (A, I): These options reduce total cost to around 10–12 million USD (a 60–70% saving compared to the current design at 31.6 million USD) while keeping displacement lower than the current value (0.013–0.019 m vs. 0.033 m).

- Balanced group (B, II): These designs cost about 19.2 million USD ($\approx 40\%$ reduction) and reduce displacement significantly to 0.006 m, more than five times smaller than the current design. They represent strong compromise solutions between cost and performance.

- Low-displacement group (C, III): These achieve the smallest displacements, about 0.003 m (almost 90% reduction compared with the current design), but with much higher costs (41–51 million USD). Such solutions are most suitable where serviceability and strict displacement limits are top priorities.

The results confirm that MOMVO explores the design space effectively and produces multiple alternatives that dominate the current design. The Pareto front offers diverse options: (i) lowest cost, (ii) balanced cost and displacement, or (iii) minimum displacement. It enables engineers and decision-makers to select the most suitable solution based on the project's requirements.

5. CONCLUSIONS AND FUTURE WORKS

The results indicate that the MOMVO–FEM framework provides an effective approach for optimizing pile-supported berthing dolphin structures under practical structural and geotechnical constraints. By integrating MOMVO with a detailed SAP2000 model, the optimization process yields a diverse set of Pareto-optimal solutions that outperform the existing design in terms of both construction cost and structural displacement. Three categories of optimal solutions are identified: low-cost designs that reduce material usage while maintaining acceptable performance, balanced solutions that achieve significant cost savings together with notable displacement reduction, and low-displacement solutions that prioritize structural stiffness at a higher cost. These alternatives demonstrate the flexibility of the proposed framework in supporting

different design objectives and decision-making needs.

Future work may extend the framework by incorporating additional objectives, more advanced soil–structure interaction models, and computationally efficient surrogate techniques, as well as by applying the methodology to other marine and offshore structures.

ACKNOWLEDGMENT

This research is funded by Vietnam Maritime University under project number: DT25-26.107.

REFERENCES

- [1] S. -J. Wu and P. -T. Chow, “Steady-state genetic algorithms for discrete optimization of trusses,” *Computers & structures*, vol. 56, no. 6, pp. 979-991, 1995.
- [2] M. R. Sierra and C. A. Coello, “Improving PSO-based multi-objective optimization using crowding, mutation and ϵ -dominance,” *Proc. of third International Evolutionary Multi-Criterion Optimization Conference*, Springer Berlin Heidelberg, pp. 505-519, 2005.
- [3] A. Kaveh and S. Talatahari, “An improved ant colony optimization for constrained engineering design problems,” *Engineering Computations*, vol. 27, no. 1, pp. 155-182, 2010.
- [4] S. Mirjalili et al, “Optimization of problems with multiple objectives using the multi-verse optimization algorithm,” *Knowledge-Based Systems*, vol. 134, pp. 50-71, 2017/10/15/ 2017.
- [5] H. Li and Q. Zhang, “Multiobjective optimization problems with complicated pareto sets, MOEA/D and NSGA-II,” *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 2, pp. 284-302, 2009.
- [6] C. C. Coello and M. S. Lechuga, “MOPSO: A proposal for multiple objective particle swarm optimization,” in *Proceedings of the 2002 Congress on Evolutionary Computation. CEC'02 (Cat. No. 02TH8600)*, 2002, vol. 2, pp. 1051-1056: IEEE.
- [7] Q. Zhang and H. Li, “MOEA/D: A multiobjective evolutionary algorithm based on decomposition,” *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712-731, 2007.
- [8] N. Khodadadi et al., “Multi-objective generalized normal distribution optimization: a novel algorithm for multi-objective problems,” *Cluster Computing*, vol. 27, pp. 10589-10631, 2024.
- [9] Ministry of Science and Technology, *TCVN 10304:2014 - Pile Foundation – Vietnamese National Design Standard*, Ministry of Science and Technology, 2014.
- [10] Ministry of Science and Technology, *TCVN 7888:2014 - Pre-Stressed Concrete Piles – Vietnamese National Design Standard*, Ministry of Science and Technology, 2014.
- [11] International Navigation Association (PIANC), *Guidelines for the design of fender systems*, PIANC, Brussels, Belgium, Working Group 33, 2002.
- [12] The Overseas Coastal Area Development Institute of Japan (OCDI), *Technical standards for port and harbour facilities in Japan*, OCIDI, 2002.